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THE TRAVELLING SALESMAN PROBLEM (TSP) A CASE STUDY

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Abstract

Ultimate goal of any industry is to make the profit, which obviously is to be maximised just by reducing the overall cost for the product. Various types of costs are associated with manufactured products. For sales promotion, salesmen have to travel from one place to another and that incurs high cost and also huge time investment. In this paper an effort is done to discuss the methods of Travelling salesman Problem (TSP) which reduces the total time or cost of travelling.

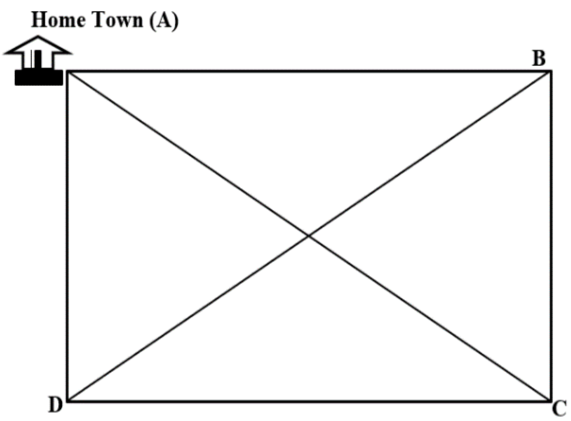
Keywords: *Hamiltonian cycle, Multigraph, Feasible solutions, Euclidean distances, Network, Dynamic Programming.*

INTRODUCTION

Marketing is the key component for any company in day to day competition. To promote and sale the products, marketing is the only and unavoidable method available. Marketing involves various sources such as Television, News Papers, Salesmen, Internet media etc. Since marketing occupies the huge part of company's budget it has always remained the crucial subject of research to study and derive techniques of reaching to the consumers at the least cost. Minimising the cost is the only available option to increase the profit margin. Marketing through salesmen is probably an only method where company as well as consumers interact personally with each other and everybody knows that no technology can replace the human approach. Thus, marketing through salesmen is the most important method among all the methods of marketing. However, it can be the most expensive and time-consuming method for marketing as number of highly paid salesmen often have to travel for hundreds of miles. Hence it is worthwhile to study the methods of reducing such travelling time and cost.

WHAT IS TRAVELLING SALESMAN PROBLEM?

To promote or to sale the products of any company, salesmen have to travel through various cities or regions frequently. In short, it becomes a network of places to be visited by salesmen. Therefore, proper planning is necessary to reduce the total time and total cost of travelling. Travelling Salesmen Problem (TSP) involves a salesman moving through a set of places or cities. The salesman starts from certain city, say the hometown and has to visit each and every city that he is supposed to visit and at the last he returns to the hometown. Main objective of the problem is to execute these activities to minimise the total length of the trip or the total travelling cost. Following is illustrative diagram of traveling routes with four places to be visited including Home town.



METHODOLOGY OF SOLVING TRAVELLING SALESMAN PROBLEM

The travelling salesman problem can be defined as under:

TSP = (G, f, t):
G = (V, E) a complete graph,
f is a function V×V →Z, t ∈ Z,
G is a graph that contains a travelling salesman tour with cost that does not exceed t.
n = $|V|$, Number of Vertices
Cij=distance from node i to node j in the graph G = (V, E).

Tour = A Hamiltonian cycle that includes every vertex exactly once.
The graph may be a directed multigraph (two arcs in opposite directions between every pair of nodes) or an undirected graph in which the distances may be symmetric i.e Cij = Cji or may not i.e Cij ≠ Cji for some i ≠ j.
If the graph is not complete, assume Cij = Cji = ∞ i.e. for the missing arcs i→j, Cij = ∞, otherwise Cij = length of the shortest path from i to j.
Set Cii = ∞, i=1, ..., n, or exclude the connection i–i in some other way.
When the total length is ∞, the Hamiltonian cycle does not exist in the given incomplete graph. The feasible solutions of the Hamiltonian cycles correspond to cyclic permutations of the vertex set.
The problem of finding a Hamiltonian cycle in a graph is Nondeterministic Polynomial (NP) complete.

TYPES OF TRAVELLING SALESMAN PROBLEM

Symmetric TSP: Cij = Cji for all i,j
Asymmetric TSP: Cij ≠ Cji for some i ≠ j
Triangle Inequality TSP Cik ≤ Cij + Cjk for all i, j, k.
Euclidean TSP: The distances between vertices are their Euclidean distances.

METHODS OF SOLVING TSP

Several Methods are available to solve the Travelling Salesmen Problem.
(1) Hungarian Method of solving assignment problem to solve TSP and
(2) Dynamic Programming method.
(3) Network Analysis Method.
Here we shall discuss the first two methods in detail.
We are familiar with the Hungarian method of solving Assignment problem. This method can also be used to solve the Travelling Salesmen Problem. First we solve the given TSP using Hungarian method to find the optimal solution. Then we have to check the TSP conditions, i.e. the salesmen starts from the initial place, visits all the places and returns to the original place without attaining the same place more than once. If the conditions are satisfied, then solution obtained by the Assignment problem (AP) will be the optimum solution for TSP also. But if not then the problem can be solved using following three methods.
Solution can be adjusted by inspection.
Form a Single Circuit
The iterative Procedure (Branch and Bound Method)

SAMPLE PROBLEM

A salesman of Sure-Cure Pharmaceutical Ltd. of California plans to visit four towns of California- Frenso, Lemoore, Coalinga and Mendota. He is at town Frenso and has to visit each town once only and returns to the town Frenso from where he started. The travelling cost (in hundred dollars) is given in the following table. Find the optimal route with an objective of minimising the travelling cost.
For the convenience of calculation, Towns Frenso, Lemoore, Coalinga and Mendota are denoted by A, B, C and D respectively.

	To				
From		Fresno (A)	Lemoore(B)	Coalinga (C)	Mendota (D)
	Fresno (A)	0	5	4	6
	Lemoore (B)	2	0	7	5
	Coalinga (C)	3	2	0	4
	Mendota (D)	4	6	8	0

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We will solve this TSP using (i) Hungarian Method of solving assignment problem and (ii) Dynamic Programming method.

Hungarian Method of solving Assignment Problem to solve TSP:
It can be seen that this is an Asymmetric TSP: Cij ≠ Cji i.e the cost of travelling between two towns may be different in different directions.
In TSP cost of travelling between same town is 0 (i.e. Travelling cost for travelling from Town A to Town A is 0 and so on). In order to solve TSP by Hungarian Method, for the convenience we will assume that such cost is very large i.e ∞, so that they can be excluded from the assignment as per the condition of TSP.

Step-I:
Subtract the smallest cost of each row from all the elements of corresponding row. We get the following form of table.

	To				
From		A	B	C	D
	A	∞	1	0	2
	B	0	∞	5	3
	C	1	0	∞	2
	D	0	2	4	∞

Step-II:
Subtract the smallest cost of each column from all the elements of corresponding column and draw the horizontal or vertical lines in such a way that all the zeros get covered in the least number of lines. As a result, we get the adjacent form of table.

	To				
From		A	B	C	D
	A	∞	1	0	0
	B	0	∞	5	1
	C	1	0	∞	0
	D	0	2	4	∞

Since number of lines so obtained are (three) lesser than number of rows/columns (four), assignment cannot be done at this stage. Therefore, follow the procedure discussed in subsequent step.

Step-III:
Select the smallest number from those numbers which are not covered by the lines, here it is 1. Now subtract this smallest number from those numbers which are not covered under the lines and add it to those elements which are located at the intersection of horizontal and vertical lines. Leave the remaining numbers as they are and draw the lines again as discussed in previous step.

	To				
From		A	B	C	D
	A	∞	1	0	0
	B	0	∞	4	0
	C	2	0	∞	0
	D	0	1	3	∞

Since four lines are obtained assignment can be done at this stage which is as under.

	To				
From		A	B	C	D
	A			*	
	B				*
	C		*		
	D	*			

Thus the path A-C-B-D-A gives the least cost which is 4+2+5+4 =15, i.e. 1500$.
While solving the TSP with Assignment Problem(AP) method sometimes alternative solutions also exists but they do not provide feasible solution for TSP, therefore such solutions are to be ignored. Sometimes in order to

reach the feasible solution we have to consider the next higher cost instead of '0' in the reduced matrix, if on considering '0' for assignment the feasible solution of TSP is is not reached.
Another method to solve the TSP is Dynamic Programing which is described below.

Dynamic Programing to solve TSP:
Let us consider the same problem with the four towns A, B, C and D.
Syntex:
$g(i,s) = \min_{K\epsilon S}\{C_{ik} + g(K, S - \{K\})\}$
Where S = {A,B,C,D}
C_{ik} = Cost of travelling from ith town to kth town.
For example: $g(A,\{B,C,D\}) = min\left\{C_{AK} + g\{K,\{B,C,D\} - \{K\}\}\right\}$
From the given travelling cost matrix following results can be derived easily.
$g(B,\emptyset)$ =2, $g(C,\emptyset)$ =3, $g(D,\emptyset)$ =4

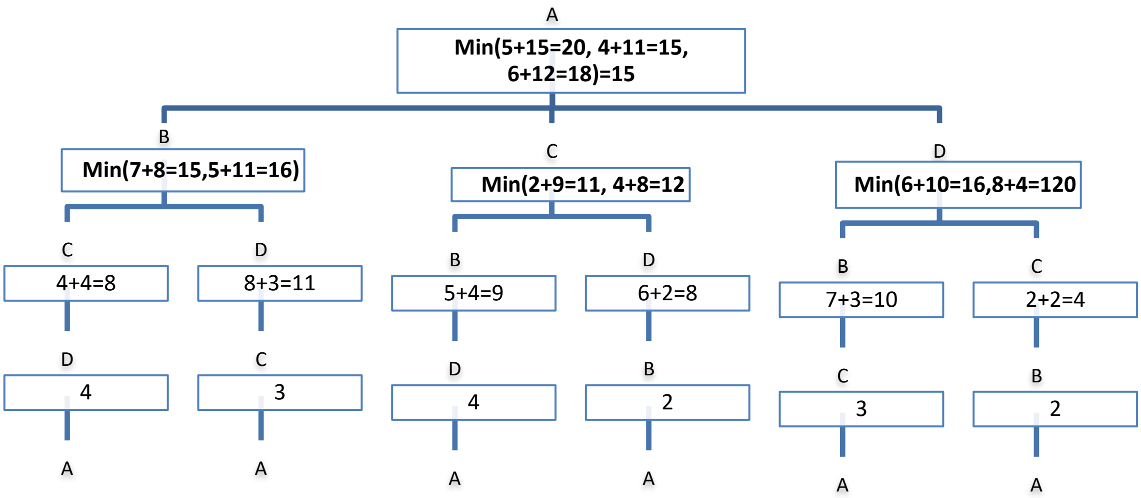
$g(B,\{C\})$ =10, $g(B,\{D\})$ =9, $g(C,\{B\})$ =4,
$g(C,\{D\})$ =8, $g(D,\{B\})$ =8, $g(D,\{C\})$ =11

$g(B,\{C,D\}) = 15$ $g(C,\{B,D\})$ =11 $g(D,\{B,C\})$ =12

$g(A,\{B,C,D\}) = \min\{C_{AB} + g(B,\{C,D\}), C_{AC} + g(C,\{B,D\}), C_{AD} + g(D,\{B,C\}\}$
= min{5+15, 4+11, 6+12}
= min{20, 15, 18}
= 15
Thus, the shortest route travelling cost is 1500 $.

DIAGRAMMATIC REPRESENTATION OF DYNAMIC PROGRAMMING

This can also be represented diagrammatically as under.



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