Financial market risk is the possibility of losing money on an investment. It is a type of danger that can result in the loss of capital to interested parties. Some financial risks include credit risk, liquidity risk, market risk, legal risk, and operational risk. Financial markets face financial risk due to various macroeconomic forces, changes to the market interest rate, and the possibility of default by sectors or large corporations. Financial risks are everywhere and come in many sizes, affecting everyone, so careful assessment of these risks will help stabilize financial industries, because understanding the dangers posed with these risks will not eliminate the risks but will help to mitigate their harm. This paper aims to review financial market risk assessment models. There are common measures of market risk including standard deviation, beta, and value at risk (VaR). Among these, VaR is the standard measure for evaluating market risk. The focus of our paper will be on methods of calculating value at risk (VaR) including traditional methods, Expected shortfall (ES) methods, Extreme Value Theory (EVT) and Copulas methods.

Key words: Financial Risk, VaR, ES, EVT, Copulas.

1. INTRODUCTION

Financial market risk is the risk of losses on financial investments caused by adverse price movements. It is one of three core risks all financial markets are required to report and hold capital against, alongside credit risk and operational risk. Market risk is classified as directional and non-directional. Directional risk is caused due to movement in stock price, interest rates and more. Non-directional risk is caused by volatility risk. Examples of market risk include changes in equity or commodity prices, interest rate moves or foreign exchange fluctuations. Some common measures of market risk include standard deviation, beta and value at risk (VaR). This paper focuses on methods of calculating VaR and ES as the standard methods for measuring market risk.

Value at risk (VaR)

Value at risk (VaR) is a statistic that quantifies the extent of possible financial losses within a firm, portfolio, or position over a specific time frame. It is a measure of the risk of loss for investments. It estimates how much a set of investments might lose, given a normal market condition, in a set time period such as a day, week, month or a year. VaR is normally measured with 95% or 99% confidence level. A VaR statistic has three main components:[2]

i. A time period
ii. A confidence level, and
iii. A loss amount (or loss percentage)

Methods of calculating VaR

VaR calculation can be done in different ways. Traditionally, there are two approaches which are divided into three methods of calculating VaR:

i. Parametric approach/analytical approach- the variance covariance method
ii. Non-parametric approach- historical simulation method and Monte Carlo simulation method

So in general, there are three methods for calculating VaR:

i. The variance- covariance method
ii. The historical simulation method
iii. The Monte Carlo simulation method

Generally VaR is calculated by:

\[ \text{VaR} = \text{[Expected Weighted Return of Portfolio} - (z\text{-score of the confidence interval} \times \text{standard deviation of the portfolio}) \times \text{portfolio value}] \]
Analytical method VaR

Analytical VaR is also known as Variance-Covariance VaR, Parametric VaR Linear VaR or Delta Normal VaR. This method works by first selecting the parameters for the holding period and confidence level. The simple formula to calculate 1-day VaR by analytical method is outlined by the following formula:

$$ VaR(\alpha) = \sigma N^{-1}(\alpha) \% \quad \text{or} \quad VaR(\alpha) = V \sigma N^{-1}(\alpha) $$

where

- $\alpha$ = the level of confidence,
- $\sigma$ = the standard deviation of changes (volatility) in the portfolio over a given time horizon,
- $V$ = the market value of the position, and
- $N^{-1}$ = the inverse function of the standard normal cumulative distribution

A prerequisite for the use of this formula is the assumption that the change in the value of the portfolio is subject to normal distribution, and the average change in the portfolio's value is zero.

For a portfolio of several assets, we need to calculate VaR of each asset from the portfolio and determine the relationship (correlation, $\rho$) between every two assets in the portfolio. Positive/negative correlation means that both assets move in the same/opposite direction, 0 means they are random. The following formula shows how calculate analytical VaR for a portfolio of $n$ assets:

$$ VaR^{2} = x \cdot \begin{bmatrix} 1 & \rho_{12} & \ldots & \rho_{1n} \\ \rho_{21} & 1 & \ldots & \rho_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{n1} & \rho_{n2} & \ldots & 1 \end{bmatrix} \cdot x^T $$

where

- $x$ = the vector of VaR of each asset in portfolio (i.e., $x = (VaR_1, VaR_2, \ldots, VaR_{n-1}, VaR_n)$)
- $\rho_{ij}$ = the correlation between the $i$th and $j$th asset

Steps to calculate VaR by Analytical method:

i) Assume distribution
ii) Compute the VaR (using the distributional assumptions) of each asset in the portfolio
iii) Compute correlation of each pair of assets
iv) Compute overall VaR

Advantages of Analytical method:

i) Historical data not needed.
ii) The simplicity of the application

Weaknesses of Analytical method:

The method is not ideal for a non-linear profile of financial instruments

The method of historical simulation (HS) VaR

The historical method applying current weights to a time-series of historical asset returns. This return does not represent an actual portfolio, but rather reconstructs the history of a hypothetical portfolio using the current position. If asset returns are all normally distributed, the VAR obtained under the historical simulation method should be the same as that under the analytical method. This method consists in the assumption that the past will be repeated. Profits and losses are sorted by size from the largest loss at one end to highest profit at the other end of the distribution.

Steps to calculate VaR by HS method:

i) Calculate daily changes in price
ii) Apply each change to actual price
iii) Sort results ascending (ranking)
iv) Select nth worst percentile

Advantages of HS method:
i) Simplicity and general applicability of the method, easy to implement
ii) It is not necessary to calculate the correlation or standard deviation
iii) Stress scenarios can be incorporated quite easily
iv) The method is not assuming a normal distribution of changes in the portfolio
v) The simple choice of the time horizon – the horizon of measurement corresponds to the length of time holding
vi) Directly gives the worst result (possible losses are simulated directly on the historical scenarios)

Weaknesses of HS method:
i) High demands for historical data, regulators generally impose 1 year of data as a minimum
ii) Short data series might not capture all the required changes
iii) Too long data series may have events not relevant to present market conditions.
iv) If an event did not occur in the sample period so it cannot be predicted
v) Changes from volatilities of the underliers can take long time before they take an effect
vi) Recent observations have the same weight as observations from the distant past

The method of Monte Carlo
This method assumes that the risk factors that affect the value of the portfolio are managed by a random process. The random process is simulated many times (e.g., 10,000 times). The result is a simulated distribution of profit and loss (P / L). The more simulations, the resulting distribution is more accurate. Monte Carlo method assumes that the share price (or value of the total portfolio) is governed by a geometric Brownian motion. Monte Carlo method is applied where a portfolio is characterized by fat tails, a portfolio that is too heterogeneous, or historical data are not available.[13]

Steps to calculate VaR by Monte Carlo method:
i) Generate random scenarios
ii) Revaluate portfolio under scenarios
iii) Sort results ascending (ranking)
iv) Select nth worst percentile

The advantages of Monte Carlo method:
i) It is the most comprehensive approach to the calculation of VaR
ii) It allows to model volatility change over time, as well as the change in average yields, fat tails
iii) The ability to include non-linear exposure risk, vega (kappa) risk (implied volatilities of the underliers)

Weaknesses of Monte Carlo method:
i) Computational complexity - the reason for the large number of risk factors and "roads"
ii) High financial cost (with respect to human capital and intellectual development)
iii) The risk of model: assumptions allocation risk factors play a key role

VaR's Strengths:
i) VaR's attractiveness lies in the fact that it is very simple to calculate
ii) It is well known and commonly used as a tool of risk management
iii) VaR exists in many forms, and that means great flexibility
iv) VaR is usable for any types of assets

But VaR's criticism:
i) The resulting VAR is only as good as the inputs and assumptions
ii) Different methods lead to different results.
iii) VaR may give a false sense of security, that there is tendency underestimate the worst results
iv) VaR does not give any information about the severity of losses beyond the VaR level
v) VaR is not a coherent risk measure

Generally, VaR does not include distribution of potential losses occurring in rare cases exceeding VaR, it does not specify the maximum size of the loss and even size less probable losses. In this case, relying only on the VaR could lead to the use of very dangerous and risky business strategies, because VaR is only an estimate and not precisely defined value. To make VaR a coherent risk measure, we need to combine it with more robust methods that give more information beyond the VaR level like, ES, EVT and Copula methods.[14]
2. LITERATURE REVIEW

Terzić and Milojević (2013) evaluated measures of market risk in circumstances of global financial crisis—empirical evidence from five countries: Serbia, Germany, Slovakia, Czech Republic, and USA. They investigated the performance of value-at-risk (VaR) models that are widely used to measure the market risk. They evaluated the performance of VaR produced by two risk models: Historical Simulation (HS) and Risk Metrics. Results indicated that during the crisis period all tested VaR models underestimated the true level of market risk exposure, showing the necessity to use more complex measures of risk such as extreme value theory, to capture and properly quantify market risk based on all relevant risk factors.\(^{[17]}\)

Adamko et al. (2015) described how the value at risk (VaR) is computed in practice and gave a short overview of VaR as one of the most essential risk measures used by many investors. They also described the basics of VaR methods and compared their similarities and differences. They found that all VaR methodologies retain the same general structure consisting of the following steps:\(^{[2]}\)

i. The calculation of the present value of the portfolio (Mark-to-Market Value), which is a function of the current values of market factors (interest rates, exchange rates and so on)

ii. An estimation of the distribution of changes in the portfolio

iii. The calculation of the VaR

Al Janabi (2007) studied how to perform value at risk analysis with Variance-covariance method. To calculate VaR using the variance/covariance (parametric) method, the volatility of each risk factor is to be extracted from a pre-defined historical observation period. The potential effect of each component of the portfolio on the overall portfolio value is then worked out. These effects are then aggregated across the whole portfolio using the correlations between the risk factors (which are, again, extracted from the historical observation period) to give the overall VaR value of the portfolio with a given confidence level. However, they suggested that, for an emerging-market environment, one needs to supplement the variance/covariance approach with other analysis such as stress testing and simulation analysis with the objective of estimating the impact of assumptions that are made under the VaR approach. Stress-testing estimates the impact of unusual and severe events on the entity’s value and should be reported daily as part of the risk reporting process. Stress-testing usually takes the form of subjectively specifying scenarios of interest to assess changes in the value of the portfolio and it can involve examining the effect of past large market moves on today’s portfolio.\(^{[3]}\)

Oppong et al. (2016) compared the performance of historical simulation and Monte Carlo simulation method in measuring VaR on ten stocks on Ghana Stock Exchange. Results indicated that, the Monte Carlo Simulation provides a better estimate than the Historical Simulation.\(^{[15]}\)

Bogdan et al. (2015) reviewed the history of the emergence of VaR methods and their usefulness in assessing the risks of financial assets. They described the three main Value at Risk (VaR) methodologies in detail: historical method, parametric method, and Monte Carlo method. After the theoretical review of VaR methods they estimated risk of liquid stocks and portfolio from the Croatian capital market with historical and parametric VaR method, after which the results were compared and explained. In their consideration, they extended the work by Sverko (2001) Risk Metrics recognized in practice as the parametric model or a model of variance and covariance. However, they concluded that VaR is not an ideal measure for risk because:

i. VaR focuses on portfolio losses but cannot fully predict future losses

ii. Using the VaR can also give false security in case if the loss is greater than the calculated

Allen et al. (2013) used EVT method to model tail-risk. They modeled the tails of daily log returns from the S&P-500, FTSE-100, CBOE-S&P Vixand FTSE-100 Volatility indices and evaluate static VaR and ES measures and compared with a dynamic two stage extreme value process with a GARCH (1,1) model to forecast daily VaR using historical data. Results indicated that EVT can be successfully applied to financial market return series for predicting static VaR, conditional VaR (CVaR) or Expected Shortfall (ES). However, the model did not work well with the implied volatility indices and produced too few violations leading to rejection of the model. So, they suggested the study to be extended using various time series models other than GARCH (1,1) with EVT to forecast VaR and ES in search of a better model.\(^{[4]}\)

Harantzis et al. (2006) conducted an empirical study of value-at-risk and expected shortfall models with heavy tails in returns using historical data. They modeled the daily returns of popular indices (S&P500, DAX, CAC, Nikkei, TSE, and FTSE) and currencies (US dollar, Euro, Yen, Pound, and Canadian dollar) for over ten years with empirical (or historical), Gaussian, Generalized Pareto (peak over threshold (POT) technique of extreme value theory (EVT)) and Stable Pareto distribution (both symmetric and non-symmetric). They Experimented on different factors that affect modeling, e.g., rolling window size and confidence level. Results indicated that, in estimating VaR, the models that capture rare events can predict risk more accurately than non-fat-tailed models. For ES estimation, the historical model and POT method proved to give more accurate estimations. However Gaussian model underestimates ES, while Stable Paretoian framework overestimates ES.\(^{[12]}\)
Bhattacharyya and Ritolia (2008) constructed a robust Value-at-Risk (VaR) measure for the Indian stock markets by combining two well known facts about equity return time series dynamic volatility resulting in the well-recognized phenomenon of volatility clustering, and non-normality giving rise to fat tails of the return distribution. Results indicated that dynamic VaR with tail estimation by Extreme Value Theory is the best method of estimating VaR at least in the Indian context. Static models are woefully inadequate in times of extreme volatility because they give extreme VaR violations. However, Dynamic measures with normality assumptions are also not good enough as they underestimate VaR. They suggested the use of Monte Carlo simulation for estimating the possible future path of dynamic volatility and the estimation of multiple periods of VaR to be taken up separately.[7]

Abad and Benito (2013) conducted a detailed comparison of value at risk estimates. In this paper they included several methods (parametric, historical simulation, Monte Carlo, and extreme value theory) and some models to compute the conditional variance. They analyzed several international stock indexes and examined two types of periods: stable and volatile periods. They found that:

i. First, all the VaR methods used in the paper seem to perform better in a stable period than a volatile period, which is a negative but significant result.

ii. Second, for a VaR method, specific results depend on the volatility model used to estimate the conditional standard deviation of the return’s portfolio.

iii. Third, the assumption regarding the distribution of the return’s portfolio seems to be important.

They concluded that the best model is a parametric model with conditional variance estimated by an asymmetric GARCH model under Student’s t-distribution of returns. As the distribution of the financial returns showed heavier tails than a normal distribution. They suggested that results may improve if the companies considered other distributions, such as the Student t-distribution.[1]

Gill (2006) developed a procedure of assessing the probability of rare and extreme events in the risk management of financial portfolios using Extreme Value Theory (EVT). EVT comprises of two approaches, the conditional approach (the BMM) and an unconditional approach (the POT). In their application POT method proved superior as it better exploits the information in the data sample in the long-term behavior rather than in short term forecasting. They computed tail risk related measures such as VaR, expected shortfall, return level and the related confidence, and applied it to daily log-returns on six market indices. Results indicated that EVT can be useful for assessing the size of extreme events.[11]

Cheng et al. (2007) presented an algorithm using copula functions to simulate random variables and further to evaluate the Value-at-Risk (VaR) of a portfolio composed of two financial assets. The copula function was used to model the dependence structure of multivariate assets. After the introduction of the traditional Monte Carlo simulation method and the pure copula method they presented a new algorithm based on mixture copula functions and the dependence measure, Spearman’s rho. This new method was used to simulate daily returns of two stock market indices in China, Shanghai Stock Composite Index and Shenzhen Stock Composite Index, and then empirically calculated six risk measures including VaR and conditional VaR. The results were compared with those derived from the traditional Monte Carlo method and the pure copula method. From the comparison they showed that the dependence structure between asset returns plays a more important role in valuating risk measures comparing with the form of marginal distributions. They also explain that, the mixture copula is a very powerful tool for risk measurement and has advantages over the other two methods in two respects;[8]

de Melo and de Souza (2004) measured financial risks with copulas method. In this paper, they focused on the unconditional distributions with the statistical modeling of the dependence structure of multivariate financial data using the concept of copulas. They compared stress scenarios constructed to those obtained using models from the extreme value theory. Results indicated that, the correct assumption and estimation of the dependence structure leads to more accurate estimation of the probabilities associated to joint gains or losses. However, they suggested more comparisons to be done in future involving copulas and models from the extreme value theory, simulating from copulas instead of simulating from the underlying multivariate distribution, improving the marginal fitting using some mixture model with emphasis on extreme tails and the use of the conditional time dependence in mean and variance using appropriate time series models followed by the fit of copulas.[9]

3. CONCLUSION AND FUTURE STUDY

In this paper we have reviewed financial market risk models. We have shown that, assessment of financial risk is vital and crucial role in the stability of financial industries. It helps to develop new methodologies that will help in eliminating or mitigating the dangers and harms caused by these risks. We have reviewed models for calculating VaR and ES. We have seen that, though VaR is the standard measure for evaluating market risk but it is not a coherent measure because it does not give information for the values beyond VaR level. Studies indicate that, ES is a coherent risk measure as it can give information beyond VaR level especially in the measure of the fat-tailed probabilities, so methods for calculating ES should be emphasized. Future work of this
study will focus on merging the Monte Carlo methods with the more robust methods that capture fat tails like, EVT and copulas to improve the performance of VaR.

4. REFERENCES